EX z transform inverse

$$x(z) = \frac{1 - 2z^{-1} + z^{-2}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} \qquad |z| > 1$$

$x[n] = ?$

$$= \frac{2\left(\frac{1}{2}z^{-2} - \frac{3}{2}z^{-1} + 1\right) + 5z^{-1} - 1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

$$= 2 + \frac{5z^{-1} - 1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

$$= 2 + \frac{8}{1 - z^{-1}} + \frac{9}{1 - \frac{1}{2}z^{-1}}$$

$$\therefore |z| > \frac{1}{2}$$

2.4 Unilateral z-transform.

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$

Both unilatural and unilatural are same if

$$x[n] = 0 \qquad n < 0$$

For unilateral ROC is always

$$|z| > |a|$$

① Time Shifting Property.

$$\mathcal{Z}\{x[n-k]\} = x[-k] + x[-k+1]z^{-1} + \ldots + z^{-k}X(z)$$

Note: If initial conditions are given then we have to utilize unilateral z-transform.

Ex: Consider a system defined by

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

And given the initial condition of

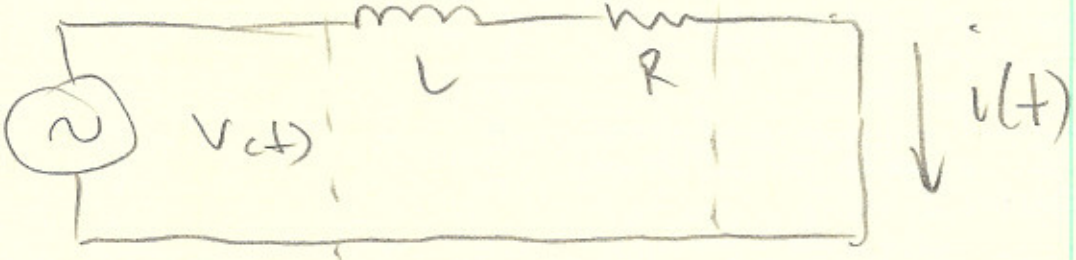
$$y[-1] = 1$$

Sol

$$Y(z) - \frac{1}{2}\left(y[-1] + z^{-1}Y(z)\right) = \frac{z}{z-1}$$

$$Y(z) = \frac{2}{1-z^{-1}} - \frac{1}{2}\frac{1}{1-\frac{1}{2}z^{-1}}$$

Ex Find the diff eqn representing the following system. Then find the T.F. impulse responce and unit step responce.



$i[-1] = 0$

$V[n] \rightarrow \boxed{h[n]} \rightarrow i[n]$

$$V(t) = Ri + L\frac{di}{dt}$$

$$V[n] = R\,i[n] + L\,\frac{i[n] - i[n-1]}{T_s}$$

let $T_s = 1$ for convience

$$V(z) = (R+L)I(z) - L\{i[-1] + z^{-1}I(z)\}$$

for TF, we must neglect initial cond.

$$\frac{I(z)}{V(z)} = \frac{1}{(R+L) - Lz^{-1}} = \frac{1}{R+L}\,\frac{1}{1 - \frac{L}{L+R}z^{-1}}$$

$$h[n] = \frac{1}{R+L}\left(\frac{L}{R+L}\right)^n u[n]$$

steady state.

then solve for initial cond. by solving for the homogeneous eqn.

$$h[n] = h_h[n] + h_{ss}[n]$$